# Understanding Fractions

Students first encounter numbers as they count —1, 2, 3, 4 . . .— and by labeling the number of objects in a set. They encounter zero (0) as describing a set with nothing in it. In this unit, students encounter another kind of number—fractions—which describe quantities that fall between whole numbers.

Although students may be familiar with the words *half, fourth*, and *quarter*, and associate these terms with parts of a whole, they may not be familiar with their mathematical meanings. Because *half* in common language does not necessarily imply the precision of the mathematical definition (e.g., think of students who want the "*bigger half*" of a cookie), students may think that *half* means part of something, but not necessarily one of two *equal* parts. The idea that the parts must be *equal* may be new to students.

When finding a fraction of a region, like a circle or a rectangle, *equal parts* means equal in area. If parts are congruent (i.e., the same shape and size), they necessarily have equal area.
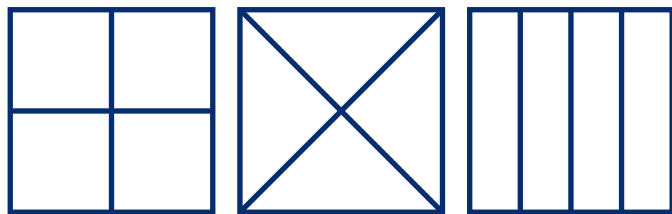
It's important to remember, though, that a region *can* be divided into non-congruent parts that have equal area. For example, in the square below, each part is equal to $\frac{1}{4}$ of the square.
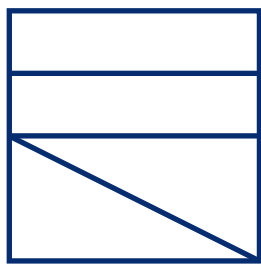
Seeing one of the triangles above as having equal area to one of the rectangles is a challenging idea for many primary students but both shapes are $\frac{1}{4}$ of the square.

Over the course of Grades 1 and 2, students work mainly with unit fractions (i.e., fractions with a numerator of 1, such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$). But they also see, describe, and create images that provide the opportunity to discuss concepts such as the following:

○ two fourths is the sum of two $\frac{1}{4}$s, and three fourths is the sum of three $\frac{1}{4}$s

○ different fractions can represent the same quantity (e.g., $\frac{2}{4}$ of a circle is equal to $\frac{1}{2}$ of a circle of the same size)
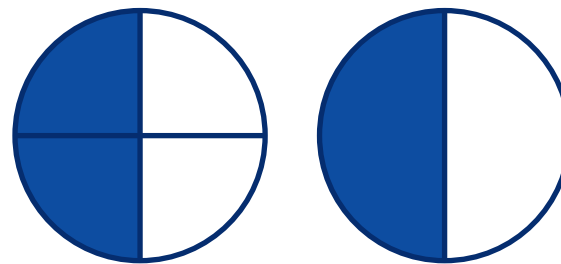
○ the whole is [two] of [two] equal shares; four fourths is equal to one whole

Students in Grades 1 and 2 are introduced to fractional notation, though using it accurately and consistently is a Grade 3 expectation. Many students, however, will likely recognize familiar fractions from seeing them used in real-world contexts. As students become familiar with the language of fractions, e.g., one half, half of, halves, etc., and hear it used to describe their work, seeing notation used as a label is a natural part of the work.

Learning about the notation used to represent equal parts of a whole can help young students—who often think the term *number* applies only to the words said while counting, or to the numerals used to label such a count—begin to understand that fractions are numbers that, most often, fall in between the counting numbers.
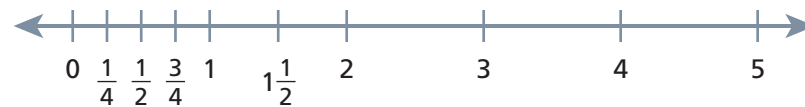
A fraction is a number that is designated by two whole numbers in relation to each other. They are obtained by dividing a whole into equal parts. The denominator (the bottom number of the fraction) indicates the number of equal parts that make up the whole; the numerator (the top number of the fraction) indicates the number of those parts that make up the quantity represented by the fraction. In the figure below, $\frac{3}{4}$ of the rectangle is shaded. The whole is divided into four equal parts (the denominator), and three of these parts (the numerator) make up the shaded quantity, $\frac{3}{4}$.
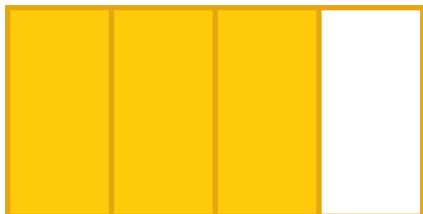
In the most general terms, mathematicians define a fraction as a number $\frac{a}{b}$, where a and b are whole numbers. The fraction $\frac{1}{b}$ is the name of 1 part of a whole that has been partitioned into b equal parts. The fraction $\frac{a}{b}$ is the quantity formed by a parts of size $\frac{1}{b}$. That is, *one fourth* $\left(\frac{1}{4}\right)$ is the name of one part of a whole that's been divided into *four equal parts. Two fourths* $\left(\frac{2}{4}\right)$ is the quantity formed by 2 parts of size $\frac{1}{4}$; *three fourths* $\left(\frac{3}{4}\right)$ is the quantity formed by 3 parts of size $\frac{1}{4}$; and *four fourths* $\left(\frac{4}{4}\right)$ is the quantity formed by 4 parts of size $\frac{1}{4}$. Such notation is not the focus of the work on fractions in Grades 1 and 2. What's important are the conceptual ideas that such notation represents.